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This project has investigated the application of wavelet methods in spread spectrum communications. Use of the wavelet transform as an alternative to the short-time Fourier and quadratic time-frequency transforms for measuring critical parameters of intercepted slow frequency hopping communication signals has been explored. Direct application of the wavelet transform was found to not offer performance advantages over the Fourier transform in this application. However, use of the wavelet transform in conjunction with Fourier methods provided an efficient hybrid framework forprecise measurement of hopping times and frequencies.

A new approach to multiple-access spread spectrum communications was also developed in this project. This approach employs orthogonal wavelets as symbols for encoding digital communication signals. Several users can transmit simultaneously in the same medium using channels defined by wavelet scale in much the same way that channels are defined by frequency in traditional frequency division multiple access. Appropriate choices of wavelet symbols make the transmitted signals robust with respect to detection and jamming.

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Report TRC-DC-9302

APPLICATION OF WAVELET TRANSFORM TECHNIQUES TO SPREAD SPECTRUM DEMODULATION AND JAMMING

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1 Summary

This project has investigated the application of wavelet methods in spread spectrum communications. Use of the wavelet transform as an alternative to the short-time Fourier transform and quadratic time-frequency transforms (e.g., the Wigner-Ville distribution) for measuring critical parameters of intercepted slow frequency hopping communication signals has been explored. The wavelet transform was found to not offer significant advantages over Fourier methods in this application because the sinusoidal analysis provided by the Fourier transform is well matched to the piecewise-sinusoidal carriers used in frequency hopping signals. However, use of the wavelet transform with sinusoidal wavelets offered an efficient multiresolution framework for precise measurement of hopping times and frequencies. A hybrid wavelet-Fourier algorithm was developed to exploit this feature.

A new approach to multiple-access spread spectrum communications was also developed under this project. This approach employs orthogonal wavelets as symbols for encoding digital information signals. Several users can transmit simultaneously in the same medium using channels defined by wavelet *scale* in much the same way that channels are defined by frequency in traditional frequency division multiple access. With appropriate choices of wavelet symbols, this approach has been shown to provide spectral spreading that makes the transmitted signals robust with respect to detection, interception, and jamming by typical methods.

2 Research Objectives

The main objectives for the research under this effort are summarized as follows:

1. Investigate the utility of the wavelet transform in estimating the spreading function for intercepted slow frequency hopping spread spectrum communication signals. Identify advantages and drawbacks of the wavelet transform compared to the short-time Fourier transform in this application.

2. Develop wavelet-based approaches to multi-user spread spectrum communications. Consider the design of wavelets that provide robustness of the transmitted signal to detection, interception, and jamming.

The results obtained in pursuit of these objectives are described in the following two sections of this report. For convenience in exposition, objective 2 is discussed first.

3 Research Results: Wavelet-Based Spread Spectrum

The purpose of this portion of the research effort was to introduce a new method of coding digital communication signals on orthogonal symbols in such a way that several users can communicate simultaneously using the same transmission medium. Such methods are known in the communications literature as code division multiple access (CDMA) schemes[1]. The two most popular CDMA schemes are frequency division multiple access (FDMA) and time division multiple access (TDMA). In FDMA, orthogonality of the various users' messages is achieved by separating them into essentially disjoint frequency bands while orthogonality of TDMA messages is obtained by constraining them to occupy disjoint time intervals.

Though FDMA and TDMA offer advantages in simplicity, they are obviously not the only ways to achieve orthogonality of transmitted message information. Other CDMA schemes were introduced originally in the context of military communications, where they were used to provide robustness with respect to detection, interception by unintended receivers, and interference – including deliberate interference (jamming) [20]. More recently the value of certain CDMA approaches in commercial applications, including cellular telephony, has been pointed out and civilian CDMA has become a topic of interest within the communications research community.

This research has developed a CDMA scheme called scale-division multiple access (SDMA) that is based on the theory of wavelets. SDMA offers much of the simplicity of FDMA while retaining many of the desirable features of more complicated CDMA schemes, including robustness to detection, interception, and jamming. In FDMA, bits are encoded on orthogonal symbols obtained by time shifting and frequency shifting a single "mother" symbol – a truncated sine wave. SDMA encodes bits on orthogonal symbols obtained by time shifting and dilating a single mother wavelet symbol, which may be chosen from a fairly broad class of wavelets. It is the separation of the various users' messages into orthogonal channels at different dilations or "scales" that suggests the name "scale division multiple access".

Closely associated with the theory of wavelets is the concept of a *frame*, which generalizes the notion of an orthonormal basis. The principle of SDMA coding has been shown to extend from orthonormal wavelet bases to wavelet frames.

[1] CDMA is interpreted in a broad sense here, i.e. as coding on orthogonal or nearly orthogonal symbols.

3.1 Mathematical preliminaries

This short section reviews some mathematical concepts and introduces some notation and conventions that will be used in explaining SDMA and in the subsequent discussion on demodulation of intercepted frequency hopping signals.

3.1.1 Lebesgue spaces

Unless otherwise noted, the expression

$$\int_S f(t)\, dt$$

will denote the Lebesgue integral of the Lebesgue measurable function $f : \mathbb{R} \to \mathbb{C}$ over the Lebesgue measurable set $S \subset \mathbb{R}$. For $1 \leq p < \infty$, the *p norm* of the Lebesgue measurable function $f : \mathbb{R} \to \mathbb{C}$ is defined as

$$||f||_p \triangleq \left(\int_{\mathbb{R}} |f(t)|^p \, dt \right)^{1/p}.$$

If the integrals of two Lebesgue measurable functions f and g on every measurable set are equal, then f and g are said to be Lebesgue equivalent. This condition is identical to the requirement that the Lebesgue measure of the set $\{t \mid f(t) \neq g(t)\}$ is zero.

For $p \geq 1$, the *Lebesgue space* $L^p \triangleq L^p(\mathbb{R})$ consists of equivalence classes of Lebesgue equivalent functions f for which $||f||_p < \infty$. With addition and scalar multiplication defined in the obvious ways, L^p is a complex Banach space for all p. Furthermore, L^2 is a complex Hilbert space with inner product

$$\langle f, g \rangle \triangleq \int_{\mathbb{R}} f(t) g^*(t) \, dt$$

One often writes "$f \in L^p$" to mean that the equivalence class of f is in L^p.

The *discrete Lebesgue space* $\ell^p \triangleq \ell^p(\mathbb{Z})$ consists of complex sequences $\mathbf{c} = \{c_k \mid k \in \mathbb{Z}\}$ for which

$$||\mathbf{c}||_p = \left(\sum_{k \in \mathbb{Z}} |c_k|^p \right)^{1/p} < \infty$$

As with L^p, ℓ^p is a complex Banach space for all p and ℓ^2 is a complex Hilbert space with inner product

$$\langle \mathbf{c}, \mathbf{d} \rangle \triangleq \sum_{k \in \mathbb{Z}} c_k d_k^*$$

Lebesgue spaces and norms over $\mathbb{R}^n$ and $\mathbb{Z}^n$ are defined analogously.

Since this report is primarily concerned with analysis in L^2, $||f||$ will be used to denote $||f||_2$ when there is no danger of confusion.

3.1.2 Fourier transform

The *Fourier transform* of $f \in (L^1 \cap L^2)$ is the function $\mathcal{F}\{f\} \triangleq \hat{f} : \mathbb{R} \to \mathbb{C}$ with values

$$\hat{f}(\omega) \triangleq \int_{\mathbb{R}} f(t) e^{-i\omega t}\, dt \tag{1}$$

Lebesgue equivalent functions clearly have the same Fourier transform. Hence the operator $\mathcal{F}$ is well defined on $L^1 \cap L^2$. It is clearly linear.

Since $L^1 \cap L^2$ is dense in L^2, the definition of $\mathcal{F}$ extends naturally to a mapping of L^2 onto L^2. The particulars of this extension are given by Plancherel's theorem [23]. One consequence of this extension is that, for functions f and g in L^2,

$$\langle \hat{f}, \hat{g} \rangle = 2\pi \langle f, g \rangle$$

which obviously implies $||\hat{f}||^2 = 2\pi ||f||^2$.

If $f \in L^2$, the *inverse Fourier transform* of $\hat{f}$ is the function $g : \mathbb{R} \to \mathbb{C}$ defined by

$$g(t) \triangleq \frac{1}{2\pi} \int_{\mathbb{R}} \hat{f}(\omega) e^{i\omega t}\, d\omega. \tag{2}$$

The function g is Lebesgue equivalent to f. Together with the preceding notes, this implies that the Fourier transform defined by (1) extends to an isomorphism of L^2 onto L^2 which is isometric except for the constant factor $\sqrt{2\pi}$.

3.1.3 Bandlimited and timelimited

In electrical engineering, it is common to regard L^2 functions as "signals" which are functions of time. Then Fourier transform variable ω is thus known as (angular) frequency.

A function $f \in L^2$ is *bandlimited* if there is an $\Omega \in \mathbb{R}$ so that $\hat{f}(\omega) = 0$ for $\omega \notin (-\Omega, \Omega)$. On the other hand, a function $f \in L^2$ is *timelimited* if there is a $T \in \mathbb{R}$ and a function g that is Lebesgue equivalent to f such that $g(t) = 0$ for $t \notin (-T, T)$.

3.1.4 Orthonormal bases of L^2

If J is an index set, a set $B \triangleq \{\phi_j \mid j \in J\}$ of elements in L^2 is orthonormal if

$$\langle \phi_j, \phi_k \rangle = \begin{cases} 1 & j = k \\ 0 & j \neq k \end{cases}$$

for all j and k in J.

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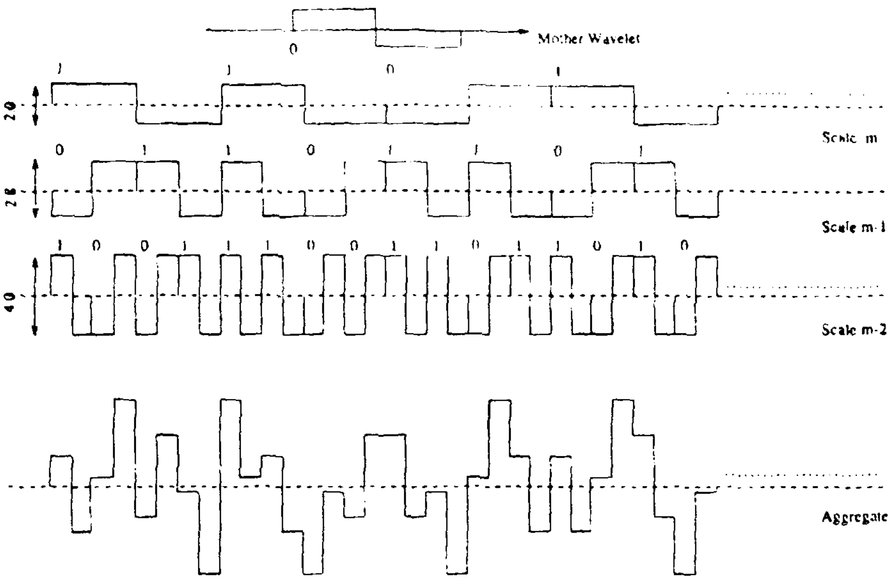

Figure 3: An example of encoding information on the Haar wavelet at different scales.

robustness with respect to detection, interference, and narrowband jamming. Design of wavelets for SDMA applications is discussed in later sections.

3.3.2 Channel capacity and synchronization

It is evident from Figure 3 that transmitters using finer scales (lower m) will be able to transmit more bits per unit time than those using coarser scales. This may be a feature in some applications; where it is not, time-division within the high-capacity channels could be used to equalize capacities.

Expressions (4) and (5) assume that the transmitters are perfectly synchronized and the propagation delays from each transmitter to the receiver are identical. Orthogonality of the channels is compromised if the received signals are not perfectly synchronized, as is evident in the example of Figure 3. This problem is not restricted to SDMA. Rather, it is inherent in any system using time-limited symbols [5]. Design of bandlimited wavelets for SDMA that achieve small channel crosstalk under asynchronous operation is discussed later in this report.

3.3.3 Coding on frames

The idea of coding on orthogonal sets can be extended to coding on frames, the extension is most well suited for transmission of large data blocks. Let $\Phi = \{\phi_j \mid j \in J\}$ be a frame and let $\tilde{\Phi} = \left\{\tilde{\phi}_j \mid j \in J\right\}$ be its dual. If $\mathbf{b} = \{b_j \mid j \in J\}$ is a collection of scalars corresponding to one information sequence as in part A of this section, the transmitter forms an analog signal s via the adjoint frame operator $\tilde{T}^*$ operating on $\mathbf{b}$

$$s = \tilde{T}^*\mathbf{b} = \sum_{j \in J} b_j \tilde{\phi}_j$$

Without noise, the receiver obtains a corresponding sequence $\beta = \{\beta_j \mid j \in J\}$ by applying the frame operator T to the received signal s.

$$\beta = Ts \quad \text{or} \quad \beta_j = \langle s, \phi_j \rangle$$

The map $T\tilde{T}^* : \mathbf{b} \to \beta$ is not generally injective, so that, an equivalence class B_β of scalar sequences is mapped to the same β. β is called the typical element of B_β.

If different information sequence corresponds to different equivalent class, then the information sequence is one to one corresponding to the typical element. In another word, the receiver can recover the information.

In the presence of additive noise z, the signal at receiver becomes $s + z$ and the demodulation operation yields

$$T(\hat{s}) = T(s + z) = \beta + Tz$$

which generally does not correspond to the equivalence class of s. To combat this, a superclass can be constructed which is the union of the equivalence classes whose typical elements are near in the sense of the norm (note T is continuous). This partitioning represents the decision mechanism.

3.4 Spectrum analysis of SDMA

The purpose of this section is to describe explicitly how the smoothed power spectral density of an SDMA signal depends on the mother wavelet and the statistical behavior of the information sequence. A procedure for pre-shaping the autocorrelation function of the information sequence is also described.

...	...	...	...	...	...	...	...
channel -1	...	b_{-1}^{-2}	b_{-1}^{-1}	b_{-1}^{0}	b_{-1}^{1}	b_{-1}^{2}	...
channel 0	...	b_0^{-2}	b_0^{-1}	b_0^{0}	b_0^{1}	b_0^{2}	...
channel 1	...	b_1^{-2}	b_1^{-1}	b_1^{0}	b_1^{1}	b_1^{2}	...
...	...	...	...	...	...	...	...

Figure 4: Information Streams

3.4.1 Power spectrum

Beginning with the form

$$v(t) = \mathrm{Re}[s(t)e^{i\omega_c t}]$$

which relates the bandpass signal v to the wide-sense stationary lowpass signal s, the autocorrelation function of v may be expressed as

$$\phi_{vv}(\tau) = \mathrm{Re}[\phi_{ss}(\tau)e^{i\omega_c \tau}]$$

where ϕ_{ss} is the autocorrelation function of the equivalent lowpass signal s. The Fourier transform of ϕ_{vv} yields the desired expression for the power spectral density Φ_{vv} in the form

$$\Phi_{vv}(\omega) = \frac{1}{2}[\Phi_{ss}(\omega - \omega_c) + \Phi_{ss}(-\omega - \omega_c)]$$

where Φ_{ss} is the power density spectrum of s. Thus it suffices to determine the autocorrelation function and the power spectral density of the equivalent lowpass signal s.

First consider the digital modulation methods for which s is represented in the general form

$$s(t) = \sum_{m,n} b_m^n W_m^n(t)$$

where b_m^n represents the n^{th} real- or complex-valued information symbol in the mth channel (see Figure 4) and W is a mother wavelet. The set $\{W_m^n \mid 2^{-m/2}W(\frac{t}{2^m} - n); n, m \in \mathbb{Z}\}$ constitutes an orthonormal wavelet set.

The autocorrelation function of s is

$$\begin{aligned} \phi(t+\tau;t) &= \frac{1}{2}\mathrm{E}[s^*(t)s(t+\tau)] \\ &= \frac{1}{2}\sum_{m',n',m,n} \mathrm{E}[b_m^{n*}, b_{m'}^{n'}]2^{-m/2}W^*(\frac{t}{2^m} - n)2^{-m'/2}W(\frac{t+\tau}{2^{m'}} - n') \qquad (6) \end{aligned}$$

Assume that the sequence of information symbols b_m^n is wide sense stationary with autocorrelation function

$$\phi_{ii}(m;n) = \frac{1}{2}\mathrm{E}[b_{m'}^{n'*}, b_{m'+m}^{n'+n}] \tag{7}$$

Then (6) can be expressed as

$$\phi(t+\tau;t) = \sum_{m',n',m,n} \phi_{ii}(m'-m;n'-n)2^{-(m'+m)/2}W^*(\frac{t}{2^m}-n)W(\frac{t+\tau}{2^{m'}}-n') \tag{8}$$

Imposing the requirement $\phi_{ii}(m;n)=0$ for $m \neq 0$ (i.e., that the information streams from different users are uncorrelated) simplifies the analysis. Then (8) becomes:

$$\begin{aligned}
&\phi(t+\tau;t)\\
&= \sum_{n',m,n} 2^{-m}\phi(0;n')W^*(\frac{t-2^m n}{2^m})W(\frac{t+\tau-2^m n'-2^m n}{2^m})\\
&= \sum_{n',m} 2^{-m}\phi(0;n')\sum_n W^*(\frac{t-2^m n}{2^m})W(\frac{t+\tau-2^m n'-2^m n}{2^m})
\end{aligned} \tag{9}$$

The summation in (9), namely,

$$\sum_n W^*(\frac{t-2^m n}{2^m})W(\frac{t+\tau-2^m n'-2^m n}{2^m})$$

is periodic in the t variable with period 2^m in the channel m. But $\phi(t+\tau;t)$ is generally not periodic (if it is, the process is called *cyclostationary*). Such a process having multiple periodicities is called a *pseudo-cyclostationary process*.

In order to compute the power spectral density of a pseudo-cyclostationary process, the dependence of $\phi_{ss}(t+\tau;t)$ on the t variable must be eliminated. This can be accomplished simply by averaging $\phi_{ss}(t+\tau;t)$ over a period within the different channels. Thus

$$\begin{aligned}
&\bar{\phi}_{ss}(\tau)\\
&= \sum_{n',m,n} \phi_{ii}(0;n')2^{-m}2^{-m}\int_{-2^m/2}^{2^m/2} W^*(\frac{t-2^m n}{2^m})W(\frac{t+\tau-2^m n'-2^m n}{2^m})\,dt\\
&= \sum_{n',m} \phi_{ii}(0;n')2^{-m}2^{-m}\sum_n\int_{-2^m/2}^{2^m/2} W^*(\frac{t-2^m n}{2^m})W(\frac{t+\tau-2^m n'-2^m n}{2^m})\,dt\\
&= \sum_{n',m} \phi_{ii}(0;n')2^{-m}2^{-m}\sum_n\int_{-2^m/2-2^m n}^{2^m/2-2^m n} W^*(\frac{t}{2^m})W(\frac{t+\tau-2^m n'}{2^m})\,dt\\
&= \sum_{n',m} \phi_{ii}(0;n')2^{-m}2^{-m}\int_{-\infty}^{\infty} W^*(\frac{t}{2^m})W(\frac{t+\tau-2^m n'}{2^m})\,dt
\end{aligned} \tag{10}$$

The Fourier transform of the relation in (10) yields the power spectral density of s in the form

$$
\begin{aligned}
\bar{\Phi}_{ss}(\omega) &= \mathcal{F}(\bar{\phi}_{ss}(\tau)) \\
&= \sum_{n',m} \phi_{ii}(0;n')2^{-2m}e^{-i\omega 2^m n'}2^{2m}|\hat{W}(2^m\omega)|^2 \\
&= \sum_{n',m} \phi_{ii}(0;n')e^{-i\omega 2^m n'}|\hat{W}(2^m\omega)|^2
\end{aligned} \tag{11}
$$

where $\hat{W}$ is the Fourier transform of W. Defining

$$\Phi_{ii}(\omega) = \sum_{n'} \phi_{ii}(0;n')e^{-i\omega n'}$$

the result in (11) can be expressed as

$$\bar{\Phi}_{ss}(\omega) = \sum_m \Phi_{ii}(2^m\omega)|\hat{W}(2^m\omega)|^2 \tag{12}$$

Defining $S(\omega) = \Phi_{ii}(\omega)|\hat{W}(\omega)|^2$ allows Equation (12) to be further simplified to

$$\bar{\Phi}_{ss}(\omega) = \sum_m S(2^m\omega) \tag{13}$$

This result illustrates the dependence of the power spectral density of s on the spectral characteristics of the mother wavelet W and the information sequence $\{b_m^n\}$. That is, the spectral characteristics of s can be controlled by design of the mother wavelet W and by design of the autocorrelation characteristics of the information sequence.

3.4.2 Autocorrelation

Since the spectrum of SDMA signal depends on the autocorrelation of information sequence, it is interesting to consider shaping of the autocorrelation.

Claim 3.1 *Suppose a random sequence*

$$\ldots \quad b_{-2} \quad b_{-1} \quad b_0 \quad b_1 \quad b_2 \quad \ldots$$

is wide-sense stationary with

$$\mathrm{E}[b_m b_n] = \begin{cases} \phi(0) & \textit{when } m = n \\ 0 & \textit{otherwise} \end{cases}$$

For real values $k_0, k_1, k_2, \ldots, k_{N-1}$, define a new sequence

$$\cdots \quad q_{-2} \quad q_{-1} \quad q_0 \quad q_1 \quad q_2 \quad \cdots$$

by

$$q_i = k_0 b_i + k_1 b_{i+1} + k_2 b_{i+2} + \ldots + k_{n-1} b_{i+N-1}$$

Then $\{b_i\}$ can be recovered from $\{q_i\}$ by knowing consecutive initial $N-1$ b_i's and the autocorrelation $\{\phi_q\}$ of the sequence $\{q_i\}$ is given by

$$\begin{bmatrix} \phi_q[0] \\ \phi_q[1] \\ \phi_q[2] \\ \vdots \\ \phi_q[N-1] \end{bmatrix} = \phi(0) \begin{bmatrix} k_0 & k_1 & k_2 & \ldots & k_{N-2} & k_{N-1} \\ k_1 & k_2 & k_3 & \ldots & k_{N-1} & 0 \\ k_2 & k_3 & k_4 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ k_{N-1} & 0 & 0 & \ldots & \ldots & \ldots \end{bmatrix} \begin{bmatrix} k_0 \\ k_1 \\ k_2 \\ \vdots \\ k_{N-1} \end{bmatrix} \tag{14}$$

and

$$\phi_q[k] = \phi_q[-k]$$

$$\phi_q[k] = 0 \quad \textit{for } k \geq N$$

The proof is omitted here.

This claim gives a method to design sequences with desired autocorrelations from $\{b_i\}$ by selecting different $k_0, \ldots, k_{N-1}$. The matrix equation (14) can also be written as

$$\begin{bmatrix} k_0 \\ k_1 \\ k_2 \\ \vdots \\ k_{N-1} \end{bmatrix} = \frac{1}{\phi(0)} \begin{bmatrix} k_0 & k_1 & k_2 & \ldots & k_{N-2} & k_{N-1} \\ k_1 & k_2 & k_3 & \ldots & k_{N-1} & 0 \\ k_2 & k_3 & k_4 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ k_{N-1} & 0 & 0 & \ldots & \ldots & \ldots \end{bmatrix}^{-1} \begin{bmatrix} \phi_q[0] \\ \phi_q[1] \\ \phi_q[2] \\ \vdots \\ \phi_q[N-1] \end{bmatrix} \tag{15}$$

So, conversely, if $\phi_q[0], \phi_q[1], \ldots, \phi_q[N-1]$ are given, $k_0, k_1, k_2, \ldots, k_{N-1}$ can be obtained iteratively as follows.

1. Initial values of k_0 to k_{N-1} are chosen.

2. These are used in the right side of (15), yielding

$$\begin{bmatrix} k'_0 \\ k'_1 \\ k'_2 \\ \vdots \\ k'_{N-1} \end{bmatrix} = \frac{1}{\phi(0)} \begin{bmatrix} k_0 & k_1 & k_2 & \ldots & k_{N-2} & k_{N-1} \\ k_1 & k_2 & k_3 & \ldots & k_{N-1} & 0 \\ k_2 & k_3 & k_4 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ k_{N-1} & 0 & 0 & \ldots & \ldots & \ldots \end{bmatrix}^{-1} \begin{bmatrix} \phi_q[0] \\ \phi_q[1] \\ \phi_q[2] \\ \vdots \\ \phi_q[N-1] \end{bmatrix}$$

3. The squared distance between $\{k_i\}$ and $\{k'_i\}$ is calculated.

$$e = \sum_{n=0}^{N-1} (k_n - k_{n'})^2$$

4. If e is less than a pre-established tolerance, $k_0, k_1, \ldots, k_{N-1}$ is the answer. Otherwise, let $k_0 = k'_0, k_1 = k'_1, \ldots, k_{N-1} = k'_{N-1}$, and return to step (2). This algorithm is not guaranteed to converge.

Note that there are many equations that can be employed instead of (15).

3.4.3 Design of the power spectrum

In this section, the performance of the SDMA scheme against jamming or exploitation attempts is discussed. Since the spectrum of the SDMA signal is dependent on the wavelet selected, one question arises naturally: "Does there exist a wavelet that causes the spectrum of the SDMA signal to look like that of white noise?". This is equivalent to the requirement that $\Phi(\omega) = \sum_m S(2^m\omega)$ is constant within a fairly wide frequency band. This requirement is vague since the scalars which m selects are unknown and the definition of "fairly wide" is vague. It is useful to discuss a related equation that gives some insights,

$$\sum_{m \in Z} S(2^m\omega) = \text{constant} \tag{16}$$

Note that m takes on all integer values in this equation. Defining $g(x) = S(2^x)$, (16) can be further written as

$$\sum_{m \in Z} S(2^m\omega) = \sum_{m \in Z} g(\log_2 \omega + m)$$

Let

$$G(x) = \sum_{m \in \mathbb{Z}} g(x+m)$$

Hence equation (16) implies G is constant for all real x. Because of the periodicity of G, it can be expanded in a Fourier series,

$$G(x) = \sum_{n \in \mathbb{Z}} c_n e^{2\pi i n x}$$

The coefficients are given by

$$\begin{aligned} c_n &= \int_{-1/2}^{1/2} G(x) e^{-2\pi i n x}\, dx = \int_{-1/2}^{1/2} \sum_{m \in \mathbb{Z}} g(x+m) e^{-2\pi i n x}\, dx \\ &= \int_{\mathbb{R}} e^{-2\pi i n x} g(x)\, dx = \hat{g}(2\pi n) \end{aligned}$$

where $\hat{g}$ is the Fourier transform of g. Hence,

$$G(x) = \sum_{n \in \mathbb{Z}} \hat{g}(2\pi n) e^{2\pi i n x}$$

The condition $\hat{g}(2\pi m) = 0$ or $\int_{\mathbb{R}} S(2^{\omega}) e^{-i\omega 2\pi m}\, d\omega = 0$ for $m \neq 0$ is necessary and sufficient for G to be constant, hence is necessary for $\sum_{m \in \mathbb{Z}} S(2^m \omega)$ to be equal to a constant.

3.5 Bandlimited orthogonal wavelet bases

The problem of channel crosstalk (intersymbol interference) that arises in the presence of asynchronous transmitter operation and propagation delay was mentioned above. One way to combat this problem is to place different transmitters' signals in disjoint frequency bands. This section derives "milti-band" wavelet bases whose elements at different scales occupy disjoint bands in the frequency domain.

One such basis is generated by the (frequency domain) mother wavelet

$$\hat{W}(\omega) = \begin{cases} 1 & \pi \leq |\omega| < 2\pi \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

With $a_0 = 2$ and $b_0 = 1$, this wavelet is easily shown to generate an orthonormal basis of L^2 [5]. This basis gives insight about how to construct other bandlimited wavelet bases in which the dilations of the mother wavelet W do not overlap in the frequency domain.

In what follows, the connected components of the support of $\hat{W}$ will be called "bands". The wavelet defined in (17) is thus a "two-band" wavelet. The following sections describe the construction of "n-band" orthonormal wavelet bases.

3.5.1 Design of n-band orthogonal sets

Suppose the support S_0 of $\hat{W}$ is the union of n disjoint intervals

$$S_0 = [s_0, s_1] \bigcup [s_2, s_3] \bigcup \ldots \bigcup [s_{n-1}, s_n]$$

Then the support of $\hat{W}_m^k$ is

$$S_m = [s_0 2^{-m}, s_1 2^{-m}] \bigcup [s_2 2^{-m}, s_3 2^{-m}] \bigcup \ldots \bigcup [s_{n-1} 2^{-m}, s_n 2^{-m}]$$

If the measure of $S_m \bigcap S_{m'}$ is zero for all non-equal integers m and m', S_0 is called an *orthogonal support*. Hence, if the support of $\hat{W}$ is orthogonal, then

$$\left\langle W_m^n, W_{m'}^{n'} \right\rangle = \frac{1}{2\pi} \left\langle \hat{W}_m^n, \hat{W}_{m'}^{n'} \right\rangle = 0 \text{ for all } m \neq m'.$$

In order that the time-shifted and dilated replicates W_m^n of W form an orthonormal set in L^2, it remains to assure that W_m^n and $W_{m'}^{n'}$ are orthogonal for $n \neq n'$ and all m. This is simplified by the observation that

$$\langle W_m^n, W_m^{n'} \rangle = \langle W_0^n, W_0^{n'} \rangle$$

Thus one must only verify that the time-shifted replicates of W are orthogonal at a single scale. The following two theorems describe how the support S_0 of $\hat{W}_0$ can be selected to ensure orthogonality.

Theorem 3.1 *Consider real numbers $0 < c_0 < c_1 < c_2 < \ldots < c_{n-1} < c_n = 2c_0$ (or $0 > c_0 > c_1 > c_2 > \ldots > c_{n-1} > c_n = 2c_0$). Then*

$$[c_0, 2c_0] = [c_0, c_1] \bigcup [c_1, c_2] \bigcup [c_2, c_3] \bigcup \ldots \bigcup [c_{n-1}, 2c_0]$$

Let $p_1, \ldots, p_n \in \mathbb{Z}$ and dilate each sub-interval $[c_j, c_{j+1}]$ by $2^{p_{j+1}}$ to form a set

$$\begin{aligned} S_0 &= [c_0 2^{p_1}, c_1 2^{p_1}] \bigcup [c_1 2^{p_2}, c_2 2^{p_2}] \bigcup \ldots \bigcup [c_{n-1} 2^{p_n}, 2c_0 2^{p_n}] \\ &= [s_0, s_1] \bigcup [s_2, s_3] \bigcup \ldots \bigcup [s_{2n}, s_{2n-1}] \end{aligned}$$

Then S_0 is orthogonal.

Theorem 3.2 *Suppose a support S_0 is orthogonal. Then S_0 can be generated by the method mentioned in Theorem 3.1 when parameters are properly chosen and possibly some subintervals are deleted.*

The proof of these theorems are given in [28].

Suppose the support of the mother wavelet W is orthogonal. Then $\{W_m^n\}$ will become an orthogonal wavelet set if and only if W also meets

$$\langle W_0^n, W_0^{n'} \rangle = 0 \text{ for } n \neq n'; n, n' \in \mathbb{Z} \tag{18}$$

This is equivalent to

$$\begin{aligned} 0 &= \langle W_0^n, W_0^{n'} \rangle \\ &= \int_{\mathbb{R}} W(t-n) W^*(t-n')\, dt \\ &= \frac{1}{2\pi} \int_{\mathbb{R}} e^{-in\omega} \hat{W}(\omega) e^{in'\omega} \hat{W}^*(\omega)\, d\omega \\ &= \frac{1}{2\pi} \int_{\mathbb{R}} e^{i(n'-n)\omega} |\hat{W}(\omega)|^2\, d\omega \end{aligned} \tag{19}$$

for all $n \neq n'$. With $k = n - n' \neq 0$, this expression becomes

$$\int_{\mathbb{R}} e^{ik\omega} |\hat{W}(\omega)|^2\, d\omega = 0 \tag{20}$$

for any non-zero integer k. Some solutions of this equation are discussed below.

3.5.2 Some bases

Suppose $\hat{W}$ is the Fourier transform of the mother wavelet W and its support on the frequency domain is symmetric with positive part is

$$V = [c_0 2^{p_1}, c_1 2^{p_1}] \bigcup [c_1 2^{p_2}, c_2 2^{p_2}] \bigcup \ldots \bigcup [c_{n-1} 2^{p_n}, c_n 2^{p_n}]$$

where $0 < c_0 < c_1 < c_2 < \ldots < c_n = 2c_0$ and $p_1, p_2, \ldots, p_n$ are integers. Also suppose that the amplitude of $\hat{W}$ equals 1 in V and that the phase is 0. Then the left side of (20) can be calculated as follows:

$$\begin{aligned} &\int_{\mathbb{R}} e^{ik\omega} |\hat{W}(\omega)|^2\, d\omega \\ &= \Big(\int_{-c_1 2^{p_1}}^{-c_0 2^{p_1}} + \int_{c_0 2^{p_1}}^{c_1 2^{p_1}} + \int_{-c_2 2^{p_2}}^{-c_1 p_2} + \int_{c_1 2^{p_2}}^{c_2 2^{p_2}} + \ldots + \int_{-c_n 2^{p_n}}^{-c_{n-1} 2^{p_n}} + \int_{c_{n-1} 2^{p_n}}^{c_n 2^{p_n}} \Big) e^{ik\omega}\, d\omega \\ &= 2\Big(\int_{c_0 2^{p_1}}^{c_1 2^{p_1}} + \int_{c_1 2^{p_2}}^{c_2 2^{p_2}} + \ldots + \int_{c_{n-1} 2^{p_n}}^{c_n 2^{p_n}} \Big) \cos(k\omega)\, d\omega \\ &= \frac{2}{k}\{-\sin(c_0 2^{p_1} k) + \sin(c_1 2^{p_1} k) - \sin(c_1 2^{p_2} k) + \sin(c_2 2^{p_2} k) - \\ &\qquad \ldots - \sin(c_{n-1} 2^{p_n} k) + \sin(c_n 2^{p_n} k)\} = 0 \end{aligned} \tag{21}$$

This equation is equivalent to the satisfaction of the following equation for any non-zero integer k:

$$- \sin(c_0 2^{p_1} k) \;\; + \;\; \sin(c_1 2^{p_1} k) - \sin(c_1 2^{p_2} k) + \sin(c_2 2^{p_2} k) -$$
$$\ldots - \sin(c_{n-1} 2^{p_n} k) + \sin(c_n 2^{p_n} k) = 0 \tag{22}$$

If $c_0 2^{p_1}$, $c_1 2^{p_1}$, $c_1 2^{p_2}$, $c_2 2^{p_2}$, ..., $c_{n-1} 2^{p_n}$, and $c_n 2^{p_n}$ satisfy (22), then $Mc_0 2^{p_1}$, $Mc_1 2^{p^1}$, $Mc_1 2^{p_2}$, $Mc_2 2^{p_2}$, ..., $Mc_{n-1} 2^{p_n}$, and $Mc_n 2^{p_n}$ will also satisfy (22) for any natural number M. So there exists a support of a "lowest" level, which will be called the *mother support*. In other words, if the end points of the support

$$V = [c_0 2^{p_1}, c_1 2^{p^1}] \bigcup [c_1 2^{p_2}, c_2 2^{p_2}] \bigcup \ldots \bigcup [c_{n-1} 2^{p_n}, c_n 2^{p_n}]$$

satisfy (22) and the end points of the support

$$V_M = [c_0 2^{p_1}/M, c_1 2^{p_1}/M] \bigcup [c_1 2^{p_2}/M, c_2 2^{p_2}/M] \bigcup \ldots \bigcup [c_{n-1} 2^{p_n}/M, c_n 2^{p_n}/M]$$

do not meet (22) for any natural number M, then V is the mother support.

Example 3.1 :

Equation (22) is difficult to solve explicitly because it is nonlinear in several variables. It can be simplified by considering the special situation where the support consists of only two symmetric intervals; i.e., the support is $[-2c_0 2^{p_1}, -c_0 2^{p_1}] \bigcup [c_0 2^{p_1}, 2c_0 2^{p_1}]$ ($c_0 > 0$). Denoting $a = c_0 2^{p_1}$, (22) becomes

$$- \sin(ak) + \sin(2ak) = 0$$

or

$$\sin(ak)[2\cos(ak) - 1] = 0 \tag{23}$$

The only valid solution of (23) is $a = m\pi$ for $m \in \mathbb{N}$ (the other solution $a = 2\pi m \pm \pi/3$ will not satisfy (23) for any non-zero integer k).

Hence, the support is $[-2m\pi, -m\pi] \bigcup [m\pi, 2m\pi]$, the mother support is $[-2\pi, -\pi] \bigcup [\pi, 2\pi]$. An orthogonal wavelet basis can be constructed whose mother wavelet is

$$\hat{W}(\omega) = \begin{cases} 1 & \text{if } \pi \leq |\omega| < 2\pi \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

Example 3.2 :

Consider the four-band case which is symmetric in the frequency domain and which has unit energy. The cut points c_0, c_1 and dilation p should meet the following condition

$$-\sin(c_0 k) + \sin(c_1 k) - \sin(c_1 2^p k) + \sin(2c_0 2^p k) = 0$$

The above equation can be decomposed into two equations (with loss of some solutions)

$$\begin{cases} \sin(2c_0 2^p k) - \sin(c_0 k) = 0 \\ \sin(c_1 2^p k) - \sin(c_1 k) = 0 \end{cases}$$

Which can be solved simultaneously to yield

$$\begin{cases} c_0 = \frac{2^p}{2^{p+1}-1}\pi \\ c_1 = \pi \end{cases}$$

Some mother wavelets are given below:

1. For $p = 0, c_0 = \pi, c_1 = \pi$

$$\hat{W}(\omega) = \begin{cases} 1 & |\omega| \in [\pi, 2\pi) \\ 0 & \text{otherwise} \end{cases}$$

2. For $p = 1, c_0 = \frac{2}{3}\pi, c_1 = \pi$

$$\hat{W}(\omega) = \begin{cases} 1 & |\omega| \in [\frac{2}{3}\pi, \pi) \bigcup [2\pi, \frac{8}{3}\pi) \\ 0 & \text{otherwise} \end{cases}$$

3. For $p = 2, c_0 = \frac{4}{7}\pi, c_1 = \pi$

$$\hat{W}(\omega) = \begin{cases} 1 & |\omega| \in [\frac{4}{7}\pi, \pi) \bigcup [4\pi, \frac{32}{7}\pi) \\ 0 & \text{otherwise} \end{cases}$$

(see [16]).

Other more sophisticated multi-band wavelet symbol constructions developed under this project are presented in [28].

4 Research Results: Frequency Hopping Signal Demodulation

Research in this area was focused on demodulation of intercepted frequency hopping spread spectrum signals in an uncluttered or lightly cluttered environment. Several slow frequency hopping signals (digital voice, text, analog audio, etc.) were generated to form a modest library of test data. This collection includes both analog and digital data and various suppressed carrier baseband modulation techniques are represented. Software for time windowed fast Fourier transform (FFT) and wavelet transform (WT) processing of spread spectrum data was written. A simple Viterbi-type algorithm that operates on raw time-frequency (periodogram) or time-scale (scaleogram) data and produces tracks of frequency hopping signals through time-frequency or time-scale space. This software was used to support empirical evaluation of WT measurements as the basis for estimating the spreading sequence of a single slow frequency hopping signal in a noise-free and clutter-free environment. Several wavelets were tested, and it became evident that the wavelets that provided scaleogram measurements that led to the best spreading sequence estimates were essentially sinusoidal. Indeed, the fact that frequency hopping signals are carried by sinusoidal waveforms seems to make Fourier analysis particularly well suited to this problem. However, in experiments, a hybrid algorithm using fixed-resolution FFT processing and multiresolution WT processing provided better spreading sequence estimates than either FFT or WT processing alone. The WT component of the algorithm provided better localization of the hopping times of the signal than the fixed resolution FFT component (with relatively long integration period chosen to provide accurate frequency measurements) than either WT or FFT processing alone. In addition, the efficiency of fast algorithms for computation of wavelet transform values at dyadically arranged points in the phase plane provided a fast approach to "focusing in" on the true hopping times and frequencies. Some additional experiments with lightly cluttered communication environments were undertaken.

Among the main reasons for the use of frequency hopping spread spectrum communication systems is their robustness with respect to detection and jamming [20]. Typical slow frequency hopping signals consist of an information signal of relatively small bandwidth modulating a carrier signal having piecewise-constant frequency that varies ("hops") within a band much larger than that of the information signal (figure 5). The spreading function is known to intended receivers,

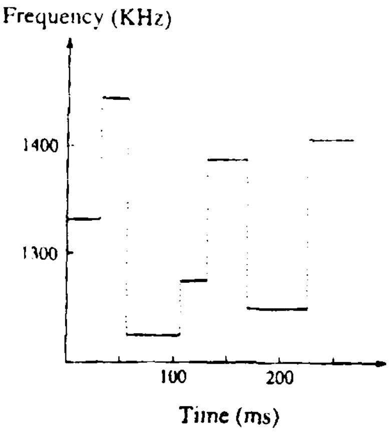

Figure 5: A typical spreading function for a slow frequency hopping communication signal. In this context, the adjective "slow" means the period between hops (which may be fixed or variable) is much longer than the duration of a symbol representing a single binary digit.

but unknown to unintended receivers who may wish to demodulate or jam the transmission. This paper presents preliminary results of research into techniques for estimating the spreading function of an unknown frequency hopping transmission using only received (intercepted) data. Since such functions are characterized by the times and frequencies associated with the hops, techniques are sought to accurately estimate these parameters. Good estimates of the spreading function are directly useful in demodulation of such signals and may also be useful in jamming them.

4.1 Time-frequency techniques

The analysis tools used in this work were the short-time Fourier transform (STFT) and the wavelet transform (WT). Other approaches for analysis of signals with time-varying characteristics, such as the Wigner-Ville and related time-frequency distributions [9] may also be applicable to this problem. These were not used in the work presented here, however.

4.1.1 The STFT

As a signal analysis tool, the primary utility of the Fourier transform is decomposition of a finite-energy signal into individual frequency components. Its value in analysis of signals with time varying characteristics, such as frequency hopping signals, is limited because information about

when the various frequency components are present in a signal is embedded in the phase of its Fourier transform in a way that is not easy to interpret. The magnitude of the Fourier transform of a frequency hopping signal, for example, contains no information about when the hops occur.

The STFT or *time-windowed* Fourier transform of a finite-energy signal $f : \mathbb{R} \to \mathbb{R}$ is a function $F : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$F(t,\omega) = \int_{\mathbb{R}} f(\tau) w(\tau - t) e^{-i\omega\tau}\, d\tau$$

In this expression, w is a bounded function, generally even and with bounded support, called a *window*. In general, window selection is based on numerous criteria [15]. In the context of frequency hopping signal analysis, the primary tradeoff is in selecting the time duration of the window. Windows of short duration allow precise temporal localization of hops at the expense of frequency resolution. Analysis using long windows sacrifices temporal resolution but improves frequency resolution.

4.1.2 The WT

The wavelet transform of a continuous-time finite energy signal f with respect to an analyzing wavelet g has values

$$W(a,b) = \frac{1}{\sqrt{a}} \int_{\mathbb{R}} f(t) g^{*}\left(\frac{t-b}{a}\right) dt$$

In this expression, g^{*} denotes the complex conjugate of g, $a > 0$ is called the dilation or *scale* parameter, and $b \in \mathbb{R}$ is the time shift parameter.

The wavelet transform has been shown to be of particular value in detecting and temporally localizing discontinuities and other sudden changes in signals [1, 22]. The ability of the WT using any given wavelet to resolve narrowband signals that are closely spaced in frequency is better at low frequencies than higher frequencies. This characteristic is ideally suited for certain applications in which frequency resolution requirements diminish in proportion to the center frequency of the band being analyzed (e.g., "constant-Q" settings). It is not desirable for analysis of frequency hopping signals, however, because the need for frequency resolution is uniform across a wide bandwidth.

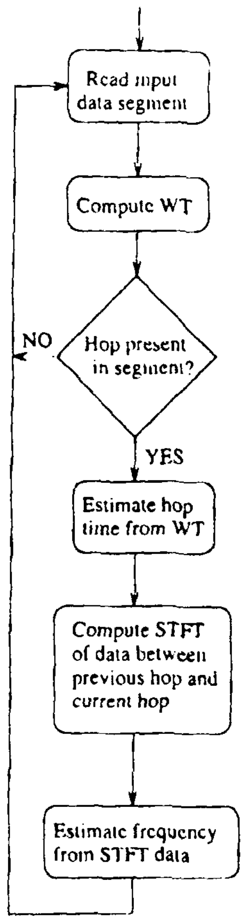

Figure 6: Flow diagram of a hybrid algorithm for estimating the spreading function of an intercepted frequency hopping signal.

4.2 An estimation algorithm

The characteristics just discussed suggest a hybrid scheme for spreading function estimation in which WT analysis is used to estimate the hopping times and STFT analysis is used to estimate the frequencies of the signal segments. Such an algorithm is depicted in figure 6.

At the first stage, the WT of a segment of the intercepted data is computed digitally. The discrete wavelet transform used at this stage is dyadic and provides one output value $w_{0,1}$ at the coarsest scale a_0. It provides two output values $w_{1,1}$ and $w_{1,2}$ at the next finer scale $a_1 = 2^{-1}a_0$. At scale $a_n = 2^{-n}a_0$, it yields 2^n values $w_{n,1}, \ldots, w_{n,2^n}$, resulting in a dyadic tree of output values ending at the finest scale $a_N = 2^{-N}a_0$. The occurrence of a hop within the segment is determined by

comparing the sequences $|w_{0,1}|, |w_{1,1}|, \ldots, |w_{N,1}|$ and $|w_{0,1}|, |w_{1,2}|, \ldots, |w_{N,2^N}|$. These sequences may be interpreted as crude local spectral estimates of the signal near the leading and trailing edges of the segment, respectively. Thus, if a hop occurs within the segment, the change in frequency structure of the signal will be reflected in the sequences. If a hop is indicated, the outputs form the finer scales are used to estimate the precise time(s) of the hop(s).

The next stage of the algorithm processes a segment of data bounded in time by two hops, as detected in the first stage of the algorithm, using a STFT with a window whose length precisely matches that of the segment. This is implemented by a zero-padded FFT [18]. The spectral estimate obtained is used to estimate the frequency of the carrier between hops.

4.3 Simulation results

A digitized frequency hopping signal with the following characteristics was produced:

- BPSK baseband modulation
- Bit rate of 9600 bps
- Hopping rate varying randomly between 50 and 250 ms
- Smallest frequency hop of 20 KHz
- No noise or interfering signals

Because the true spreading function of this signal was known, it was possible to reconstruct the baseband signal from the frequency hopping signal simulating the reconstruction possible under ideal conditions at an intended receiver. The signal-to-noise ratio (SNR) obtained in this reconstruction was approximately 25 dB.

The hybrid algorithm described above was used to estimate the spreading function of the simulated signal. The parameters of the algorithm were chosen based on the assumption that the shortest time between hops would be no less than 25ms and that the smallest frequency hop would be no less than 10 KHz. Otherwise, the algorithm used no specific *a priori* knowledge about the signal. The baseband signal was reconstructed using the estimated spreading function with a SNR of approximately 14 dB.

A similar spreading function estimation algorithm using STFT rather than WT to estimate the hopping times was implemented and run on the same data. It was found to be much more computationally efficient, but the estimated hopping times were not as accurate as those obtained using WT. This loss of accuracy reduced the reconstruction SNR to approximately 8 dB.

4.4 Demodulation versus jamming

In principle, accurate estimation of the spreading function of a frequency hopping signal can be useful in jamming as well as in demodulation. For jamming applications, a hop must be detected and the frequency of the new segment estimated essentially immediately in order for the jamming transmitter to switch to the new frequency before the next hop. Thus, estimation algorithms must be causal and very fast for this application. On the other hand, the accuracy of the estimate may not be crucial. Just having a general idea where in the phase plane the signal is concentrated will allow much more efficient use of jammer energy than constant jamming of a broad frequency band.

For demodulation of an intercepted signal neither causality nor fast algorithm execution may be necessary. Latencies of seconds or even minutes may have little effect on the utility of the spreading function estimate in demodulation. However, accuracy of the estimate in this application is crucial.

5 Publications

Results from this research effort have been published as follows.

5.1 Journal and conference proceedings papers

The following papers describing results obtained under this effort were published or submitted for review and possible publication:

1. N. Bhouri and D. Cochran, "Multiscale Time-Frequency Techniques for Spread Spectrum Demodulation and Jamming," *Proceedings of the Twenty-Sixth Asilomar Conference on Signals, Systems, and Computers*, October 1992.

2. D. Cochran and C. Wei, "Scale Based Coding of Communication Signals," *Proceedings of the IEEE-SP International Symposium on Time-Frequency and Time-Scale Analysis*, October

1992.

3. D. Cochran and S. Enserink, "Use of Cyclostationary Features in Frequency Hopping Signal Segment Association," *Proceedings of the Workshop on Cyclostationary Signals*, August 1992.

4. C. Wei and D. Cochran, "Construction of Discrete Orthogonal Wavelet Bases," *Record of the IEEE International Symposium on Information Theory*, January 1993.

5. D. Cochran and C. Wei, "Scale-Division Multiple Access – Part I: Wavelet Coding and Spread Spectrum," Submitted to *IEEE Transactions on Information Theory.*

5.2 Theses

The following thesis describing results obtained under this effort was prepared and defended:

- N. Bhouri, *Application of Multiresolution Time-Frequency Techniques to Spread Spectrum Demodulation and Jamming*, M.S. Thesis, Arizona State University, December 1991.

An additional thesis is in preparation and is expected to be defended in May, 1993:

- C. Wei, *Scale-Division Multiple Access*, M.S. Thesis, Arizona State University, May 1993 (in preparation).

6 Participating Personnel

Personnel contributing to this research effort were:

1. D. Cochran, Principal Investigator
2. C. Wei, Research Assistant
3. N. Bhouri, Research Assistant
4. D. Sinno, Research Assistant

The research component of the following graduate degree was undertaken as part of this effort:

1. N. Bhouri, Master of Science in Electrical Engineering, December 1991. Thesis: *Application of Multiresolution Time-Frequency Techniques to Spread Spectrum Demodulation and Jamming*

2. C. Wei, Master of Science in Electrical Engineering, ~~to~~ complete May 1993. Thesis: *Scale-Division Multiple Access*

7 Research Interactions

The following research interactions were related to this project:

1. An oral presentation entitled "Application of Multiscale Processing to Characterize Frequency Hopping Spread Spectrum Signals" was prepared by D. Cochran and N. Bhouri presented by D. Cochran at the AFOSR/AFIT Symposium on Application of Wavelets to Signal Processing at Wright-Patterson AFB in March 1991.

2. D. Cochran attended a series of meetings at Wright Laboratories in October 1991 to discuss this project and related topics with the AFOSR Program Manager (J. Sjogren), Air Force Laboratory personnel (J. Stephens, L. Gutman, J. Tsui), and AFIT Faculty (M. Oxley, G. Warhola). In particular, a progress briefing on this project was presented to the Program Manager at this time.

3. D. Cochran presented an invited lecture in the AFIT/AFOSR Distinguished Lecture Series at the Air Force Institute of Technology in September 1992. During this visit to Wright Patterson AFB, he collaborated extensively with AFIT faculty (M. Oxley, B. Suter) and met with Wright Laboratories personnel.

4. D. Cochran presented talks on this project at (a) the University of Arizona Electrical Engineering Department Colloquium [35 attendees] in November 1990, (b) the IEEE Signal Processing and Communications Society Phoenix Chapter Meeting [48 attendees] in February 1991, (c) the Motorola Government Electronics Group Signal Processing Seminar Series [13 attendees] in April 1991, and (d) MIT Research Laboratory for Electronics Signal Processing Group Seminar [12 attendees] in February 1992.

8 Inventions and Patents

No inventions or patent disclosures resulted from this research program.

9 Additional Information

Work on the most promising aspects of this project is continuing under grant No. F449620-93-1-0051.

References

[1] M. Basseville, "Detection of abrupt changes in signal processing." In *Wavelets: Time-Frequency Methods and Phase Space Methods*, J.M. Combes, A. Grossman, and Ph. Tchamitchanian, Eds., pp. 99-101, Springer-Verlag, 1989. See also the references given in this note.

[2] N.H. Bhouri, *Application of Multiresolution Time-Frequency Techniques to Spread Spectrum Demodulation and Jamming.* M.S. Thesis, Arizona State University, 1991.

[3] N.H. Bhouri and D. Cochran, "Multiresolution time-frequency techniques for spread spectrum demodulation and jamming," *Proceedings of the 26th Asilomar Conference on Signals, Systems, and Computers,* vol. 1, pp. 105-107, October 1992.

[4] P.J. Burt and E. H. Adelson, "The Laplacian pyramid as a compact image code," *IEEE Transactions On Communications,* vol. COM-31, no. 4, pp. 532- 540, April 1983.

[5] D. Cochran, *Notes on Mathematical Signal Analysis.* Telecommunications Research Center Report No. TRC-DC-9201, Arizona State University, 1992.

[6] D. Cochran and N. Bhouri, "Application of multiscale processing to characterize frequency hopping spread spectrum signals," *AFOSR/AFIT Workshop on Wavelet Applications in Signal Processing,* March 1991.

[7] D. Cochran, R. Hedges, R. Martin, J. Quirin, and C. Wei *Lectures on Time-Frequency Signal Analysis.* Telecommunications Research Center Report No. TRC-DC-9301 Arizona State University, 1993.

[8] D. Cochran and C. Wei, "Scale based coding of digital communication signals," *Proceedings of the IEEE-SP International Symposium on Time-Frequency and Time-Scale Analysis,* October 1992.

[9] L. Cohen, "Time-frequency distributions: A review," *Proceedings of the IEEE,* vol. 77(7), pp. 941-981, July 1988.

[10] I. Daubechies, *Ten Lectures on Wavelets.* SIAM Press, 1992.

[11] I. Daubechies, "The wavelet transform, time-frequency localization and signal analysis," *IEEE Transactions on Information Theory*, vol. IT-36(5), pp. 961-1005, September 1990.

[12] I. Daubechies, "Orthonormal bases of compactly supported wavelets," *Communications on Pure and Applied Mathematics*, vol. XLI, pp. 909- 996, 1988.

[13] I. Daubechies, A. Grossmann, and Y. Meyer, "Painless nonorthogonal expansions," *J. Math. Phys.* vol. 27, no. 5, pp. 1271-1283, 1986.

[14] G.B. Folland, *Real Analysis: Modern Techniques and Their Applications.* Wiley, 1984.

[15] f. harris, "On the use of windows for harmonic analysis with the discrete Fourier transform," *Proceedings of the IEEE*, vol. 66(1), January 1978.

[16] S.G. Mallat, *Multiresolution Representations and Wavelets.* Ph.D. Thesis, University of Pennsylvania, August 1988.

[17] D.L. Nicholson, *Spread Spectrum Signal Design, LPE & AJ Systems.* Engineering Research Associates, Inc., 1988.

[18] A.V. Oppenheim and R.W. Schafer, *Discrete-Time Signal Processing*, Prentice-Hall, 1989.

[19] A.V. Oppenheim and G.W. Wornell, "Representation synthesis and processing of self similar signals," *AFIT/AFOSR Workshop on Wavelet Applications in Signal Processing*, March 1991.

[20] R.A. Pickholtz, D.L. Schilling, and L. B. Milstein, "Theory of spread spectrum communications a tutorial," *IEEE Transactions on Communications*, vol. COM-30(5), pp. 855-884, May 1982.

[21] J.G. Proakis, *Digital Communications.* McGraw-Hill Book Company, 1983.

[22] O. Rioul and M. Vetterli, "Wavelets and signal processing," *IEEE Signal Processing Magazine*, vol. 8(4), pp. 14-38, October 1991.

[23] W. Rudin, *Real and Complex Analysis*, McGraw-Hill, 1974.

[24] R.A. Scholtz, "The origins of spread spectrum communications," *IEEE Transactions on Communications*, vol. COM-30(5), pp. 822-854, May 1982.

[25] B.W. Suter and M. E. Oxley, "On variable overlapped windows and weighted orthonormal bases," to appear in: *IEEE Trans. on Signal Processing.*

[26] F.B. Tuteur, "Wavelet transformations in signal detection," In *Wavelets: Time-Frequency Methods and Phase Space Methods*, J.M. Combes, A. Grossman, and Ph. Tchamitchanian, Eds., pp. 132-138, Springer-Verlag, 1989.

[27] C. Wei and D. Cochran, "Construction of discrete orthogonal wavelet bases," *Proceedings of the IEEE International Symposium on Information Theory*, pp. 330, January 1993

[28] C. Wei, *Scale-Division Multiple Access*, M.S. Thesis, Arizona State University, May 1993 (in preparation).

[29] G.W. Wornell, "Communication over fractal channels," *Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 3, pp. 1945-1948, May 1991.

[30] G.W. Wornell, *Synthesis, Analysis, and Processing of Fractal Signals*. Ph.D Thesis, Massachusetts Institute of Technology, 1991.

[31] G.W. Wornell and A.V. Oppenheim, "Wavelet-Based representations for a class of self-similar signals with application to fractal modulation," *IEEE Transactions on Information Theory*, vol. IT-38(2), pp. 785-800, March 1992.